Name:			
	HL Applications&Interpretation Unit1 SOLUTIONS + MARKSCHEME **Sequences, Logs, Scientific notation, Upper & lower bounds and percentage error, Introducing 'e', Amortization, Least square's regression & Pearson's correlation coefficient(r), Rates of change(differences method)**		
	Unit1: **50 marks**	Time:	55 minutes

Q1) [10marks]

Below is the data comparing the length of people's forearms, and the length of their middle fingers, with their height.

Middle Finger(mm)	95	78	83	78	90	74	84	79	79	85	81	67	80	90	73
Height(cm)	180	149	175	168	186	163	170	167	166	189	165	155	175	170	148

Forearm(cm)	26	23	24	28	27	25	26	25	25	26	27	22	26	23	25
Height(cm)	174	169	166	188	179	176	185	163	156	160	170	156	169	162	169

(i) Calculate Pearson's correlation coefficient for each of the two data sets above and state which, length of forearm or middle finger, you think is the better predictor of someone's height and give a reason.

Middle finger **r = 0.739 (to 3 s.f.)** (A1) (unrounded) A1 (rounded to 3 s.f.)
Forearm **r = 0.651 (to 3 s.f.)** (A1) (unrounded) A1 (rounded to 3 s.f.)
Middle finger has a slightly higher correlation coefficient suggesting it is a slightly better predictor of a person's height than the length of their forearm **A1**

Middle Finger & height

Linear Regression (mx+b)

X List: 'midfingermm
Y List: 'h1
Save RegEqn to: f1
Frequency List: 1
1st Result Column: c[]

RegEqn	m*x+b
m	1.22363
b	69.20472
r^2	0.546239
r	0.73908

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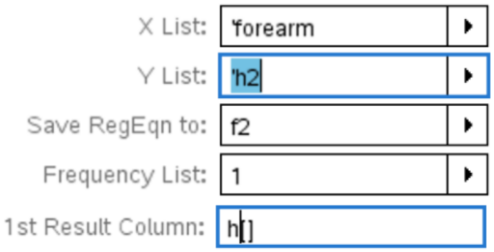
Forearm & height

Linear Regression (mx+b)



RegEqn	m*x+b
m	3.7917
b	73.916
r^2	0.424101
r	0.65123

(ii) Using the data you chose in part (i), calculate the Least squares regression line for the data

Calculator computes the least squares regression line at the same time as the 'r' value – see example screenshots from TiNspire above (Casio etc. also calculate both at same time):

Height(cm) = 1.22373 x middle finger length + 69.20472 => **h = 1.22m + 69** (to 3s.f.) **A1 A1**
(where h=height (cm) and m = middle finger length (mm) **award only A1** if not **gradient & y-intercept not rounded to 3 s.f.**

[Height(cm) = 3.7917 x forearm length(cm) + 73.916 => **h = 3.79f + 73.9** (to 3s.f.)] – award full "follow through" marks if candidate continues with incorrect model from part (i)]

(b) use the regression equation found in (ii) to predict the height of someone with a middle finger of length:

(iii) 83mm **1.22(83) + 69 = 170.26cm ≈ 170cm** (to 3 s.f.) **A1**
[ft marks if used unrounded gradient & intercept from (ii)]

(iv) 95mm **1.22(95) + 69 = 184.9cm ≈ 185cm** (to 3 s.f.) **A1**
[ft marks if used unrounded gradient & intercept from (ii)]

(v) Which prediction do you think is likely to be more accurate, give a reason.
83mm prediction because there are many more data points around this value, which have contributed to the calculation of the linear least squares model, than there are around the 95mm length (which is the maximum value in the original data, the next closest point being at 90mm) i.e. 83mm is definitely **inter**polation, 95mm, being at the extreme, the pattern may change thereafter (risk/borderline **extra**polation).

[5]

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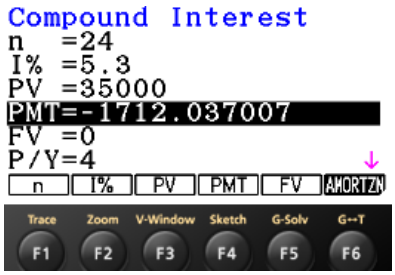
Q2) [8 marks]
You decide to take out a loan to buy Tesla's Standard Model Plus at 35 000€ over 6 years at an interest rate of 5.3%, compounded quarterly.

(a) Calculate your monthly repayments

[2]

Casio:



TiNspire:

tvmPmt(N, I, PV, FV, [PpY], [CpY], [PmtAt]) = **tvmPmt(24,5.3,35000,0,4,4) = - 1 712.04€ A1 A1**

[**Note:** PmtAt is interest calculated at the BEGINning or END of each time period (END is automatically used so no need to enter anything here unless question specifically asks for "beginning" - **which won't be the case for IB exams**]

Common mistake:
tvmPmt(72,5.3,35000,0,12,12) = -568.556€ -> misunderstood that repayments are **always** monthly, **regardless** of **compounding periods: yearly, quarterly, semi-annually, weekly etc.**

(b) At the end of 3 years, how much of the capital have you paid back?

TiNspire:

Method1: Principal paid
∑Prn(NPmt1, NPmt2, N, I, PV, [Pmt], [FV], [PpY], [CpY], [PmtAt], [roundValue])

$$\Sigma Prn(1,12,24,5.3,35000,-1712.04,0,4,4,,2)$$
$$-16120.8$$

∑Prn (1,36,72,5.3,35000,-1712.04,0,12,12,,2)

OR

Method2: amortization table

amortTbl(NPmt, N, I, PV, [Pmt], [FV], [PpY], [CpY],,dp)
amortTbl(24,24,5.3,35000,-1712.04,0,4,4,,2) : 35 000 – 18 879.20 = 16 120.80€
(M1) for the subtraction A1 answer

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mth	capital	interest	balance
0	0.	0.	35000.
1	-463.75	-1248.29	33751.7
2	-447.21	-1264.83	32486.9
3	-430.45	-1281.59	31205.3
4	-413.47	-1298.57	29906.7
5	-396.26	-1315.78	28590.9
6	-378.83	-1333.21	27257.7
7	-361.16	-1350.88	25906.9
8	-343.27	-1368.77	24538.1
9	-325.13	-1386.91	23151.2
10	-306.75	-1405.29	21745.9
11	-288.13	-1423.91	20322.
12	-269.27	-1442.77	18879.2
13	-250.15	-1461.89	17417.3
14	-230.78	-1481.26	15936.1
15	-211.15	-1500.89	14435.2
16	-191.27	-1520.77	12914.4
17	-171.12	-1540.92	11373.5
18	-150.7	-1561.34	9812.13
19	-130.01	-1582.03	8230.1
20	-109.05	-1602.99	6627.11
21	-87.81	-1624.23	5002.88
22	-66.29	-1645.75	3357.13
23	-44.48	-1667.56	1689.57
24	-22.39	-1689.65	-0.08

OR

Method3: Balance

bal(up to which payment, total number of payments over whole loan, interest rate, loan amount, -monthly repayment, final value of loan (=0 as all paid back), payment periods per year, compounding periods per year,, n° of decimal places to round to):

bal(12,24,5.3,35000,-1712.04,0,4,4,,2)

18879.2

35 000 – 18 879.20 = 16 120.80€

[2]

(c) What percentage of your 20th re-payment is used to pay back the interest on the loan?

Method1: Interest calculate command

ΣInt(start payment, last payment, total n° payments,interest rate, loan amount, monthly repayment, payPerYear,CompoundedPerYr,,Rounding)

ΣInt(20,20,24,5.3,35000,-1712.04,0,4,4,,2)

-109.05

Method2: Amortization Table

mth	capital	interest	balance
20	-109.05	-1602.99	6627.11

$\frac{-109.05}{-1712.04} \times 100 = 6.36961\%$ Answer: **6.37% (to 3 s.f)**

(M1) correct fraction seen anywhere A1 correct answer (to 3 s.f.)

COMMON MISTAKE: compounded monthly instead of quarterly

19	-120.41	-448.15	26813.8
20	-118.43	-450.13	26363.7
21	-116.44	-452.12	25911.6

[2]

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The car is advertised as having an autonomy (before needing to recharge) of 275km, plus or minus an error of 9.5%.

(d) Calculate the upper and lower bound of the car's autonomy.

Lower bound = (1 – 0.095) x 275 = 248.875 km ≈ **249 km (to 3 s.f.)** **A1**
Upper bound = (1 + 0.095) x 275 = 301.125 km ≈ **301 km (to 3 s.f.)** **A1**
249 km ≤ autonomy ≤ 301 km

[2]

Q3) [4 marks]
The Sun looks the brightest star in the sky due to its proximity to Earth. **Sirius**, with an 'apparent' magnitude (m) of only -1.44, compared to the Sun's -26.7, is actually the brightest star that is visible to the naked eye. Sirius is approximately 2.64 parsecs from Earth and the Sun only 4.848×10^{-6} parsecs.
The following formula converts 'apparent' magnitudes (m) to '**absolute'** magnitudes (M) (measures of the total energy emitted by a star), where d = distance, in parsecs, from Earth (1 parsec ≈ 3.26 light years).

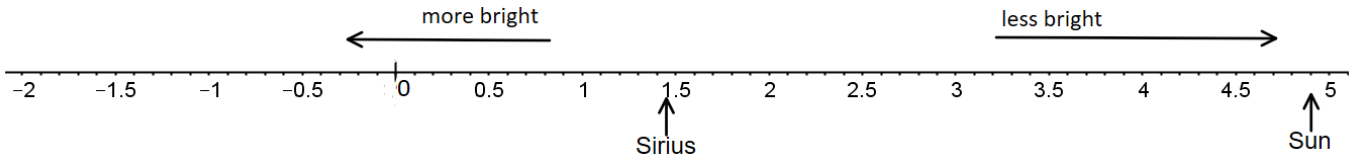
$$M = m - 2.5\,Log\left[\left(\frac{d}{10}\right)^2\right]$$

(a) Use this formula to find the values of *a* and *b* in the below equation and hence, or otherwise, calculate how many times brighter the absolute magnitude of Sirius is compared to the Sun.

$M = m - 5\,Log\left[\left(\frac{d}{10}\right)\right]$ = m – 5 [Log(d) – Log(10)]
= m – 5Log(d) + 5Log(10) = **m – 5Log(d) + 5(1)**
a = -5 **b = 5** **A1 A1**

[2]

(ii) The 'absolute magnitude' measure of brightness is a logarithmic scale. Negative magnitudes are **brighter** than positive magnitudes. A magnitude of 1 is 2.512 times greater than a magnitude of 2. How many times brighter is Sirius than the Sun in 'absolute' terms?



If a magnitude of 1 is 2.512 times brighter than a magnitude of 2 this means that $2.512^{\text{difference in magnitude}}$ = difference in 'absolute' brightness.

$$-1.44 + 5Log(2.64) + 5 = 1.451980366 \approx 1.45(to\ 3s.f.)$$

$$-26.7 + aLog(4.848 \times 10^{-6}) + b = 4.872186944 \approx 4.87(to\ 3s.f.)$$ **M1 both** magnitudes correct

$2.512^{4.87-1.45} \approx 23.33818\ldots$ (or 23.34263 if unrounded values used)
≈ **23.3 times brighter** (to 3 s.f) **A1**

[2]

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Q4)

Q4) [10marks]

A money management book, 'Financing Freedom', recommends saving $50 on the 1st day of your first month of work, $60 in your second, increasing by $10 every month on the 1st of each month during your first year in paid employment. Hua decides to follow this advice and invests each months savings in an account that pays 4% interest per year (compounded monthly), he leaves it in the account for 5 years before removing it all, ready to help her buy her first car. Give **all answers to 2 decimal places.**

(i) Calculate the total amount of money Hua saves in her first year of work. [2]

(ii) does not include interest

(iii) How much money in total will Hua have in her account at the end of the 5 years. [4]

savings + interest

Zhang Wei decides to do the same for the first year of his paid employment, but saves $80 the first month, increasing this by '$d' each month to the end of the year.

At the end of the 5 years, investing each months savings at 4% interest per year (compounded yearly), Zhang Wei savings are worth $11 057.10.

(b) Determine how much Zhang Wei increased his savings by each month. [4]

Zhang wei's savings + interest =

AP, GP

$U_1 = 50$ $u_1 = 50$

$U_2 = 60$ $d = 10$

$U_3 = 70$ $n = 5 \times 12 = 60$

etc.

1st Jan = 50

1st Feb = 60

(i) $S_{12} = \frac{n}{2}[U_1 + U_n]$

$= \frac{12}{2}[50 + u_1 + (n-1)d]$

$50 + 11(10)$

$= 6[50 + 160]$

$6 \times 210 = 1260$

(ii) 5 years = 60 months

$\$50 \times (1 + \frac{4\%}{12})^{60} = 50 \times [1 + \frac{0.04}{12}]^{60}$ ← 1st payment

$\$60 \times (1 + \frac{0.04}{12})^{59} = 60 \times [1 + \frac{0.04}{12}]^{59}$

See next page for the rest of this solution to (ii)

Calculate each month's interest over 5 years

Store in calculator memory (sto→) and sum

$50 \times [1 + \frac{0.04}{12}]^{60}$ sto→ A

CTRL+C

CTRL+V

$60 \times [1 + \frac{0.04}{12}]^{59}$ sto→ B

$[50 + 11(10)] \times [1 + \frac{0.04}{12}]^{49}$

$160 \times [1 + \frac{0.04}{12}]^{49}$ sto→ L

$A + B + C + D + E + \ldots + L$

$=$

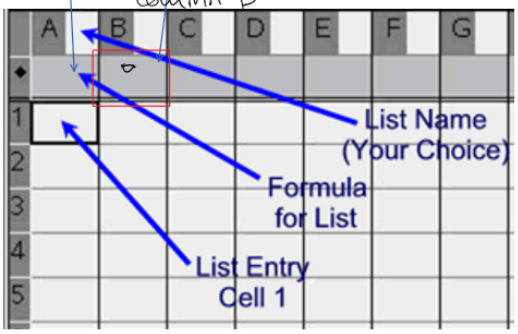
Method 2: Spreadsheet with Formulae

$U_1 + (n-1)d$

	A	B
1	1	
2	=A1+1	
3		
4		
5		
6		

$= [50 + (A1-1)10] \times [1 + \frac{0.04}{12}]^{[60-(A1-1)]}$

automatically generates the values using this formula in all cells of column B



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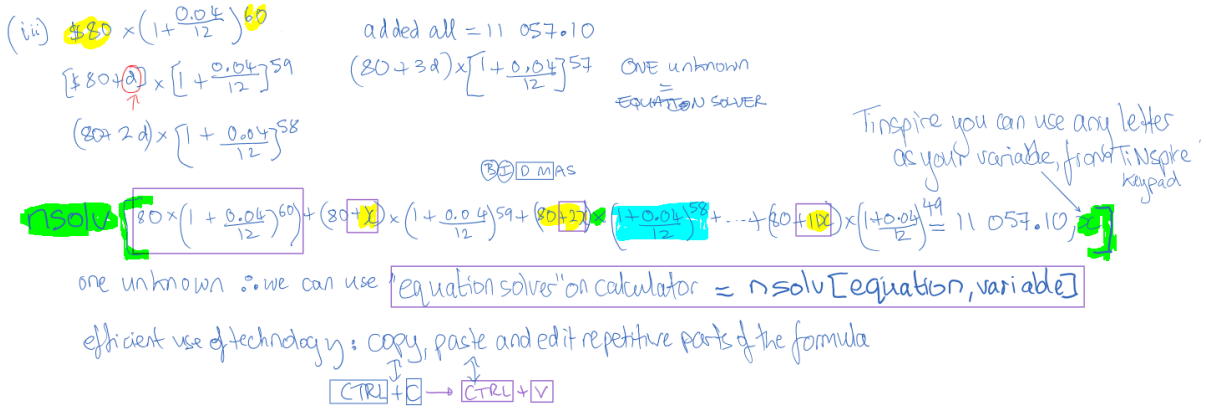


[4]

Q5) [10marks]

Stars visible in the night sky come in many shapes and sizes. To start classifying these, their 'brightness', measured in "flux" (Watts/m²), is compared to the brightness of the star Vega. Vega is assigned an **'apparent magnitude' (m)** of zero.

(i) If we used the equation below, **show that** Vega has a brightness equal to 10^{-11} flux (watts/m²):

$$\mathbf{m = - 27.5 - Log(f^{2.5})}$$

Laws of Logs: (0) = - 27.5 - 2.5 Log(f) **M1** for setting up the correct equation and use of law of logs

therefore $\frac{-27.5}{2.5} = Log(f)$ therefore: 10^{-11} = f **A1** both **-11** (or equivalent) **= Log(f)** and **10^{-11} = f** must be seen.

[2]

(ii) Sketch the above equation on the grid below, labelling the graph with its correct equation

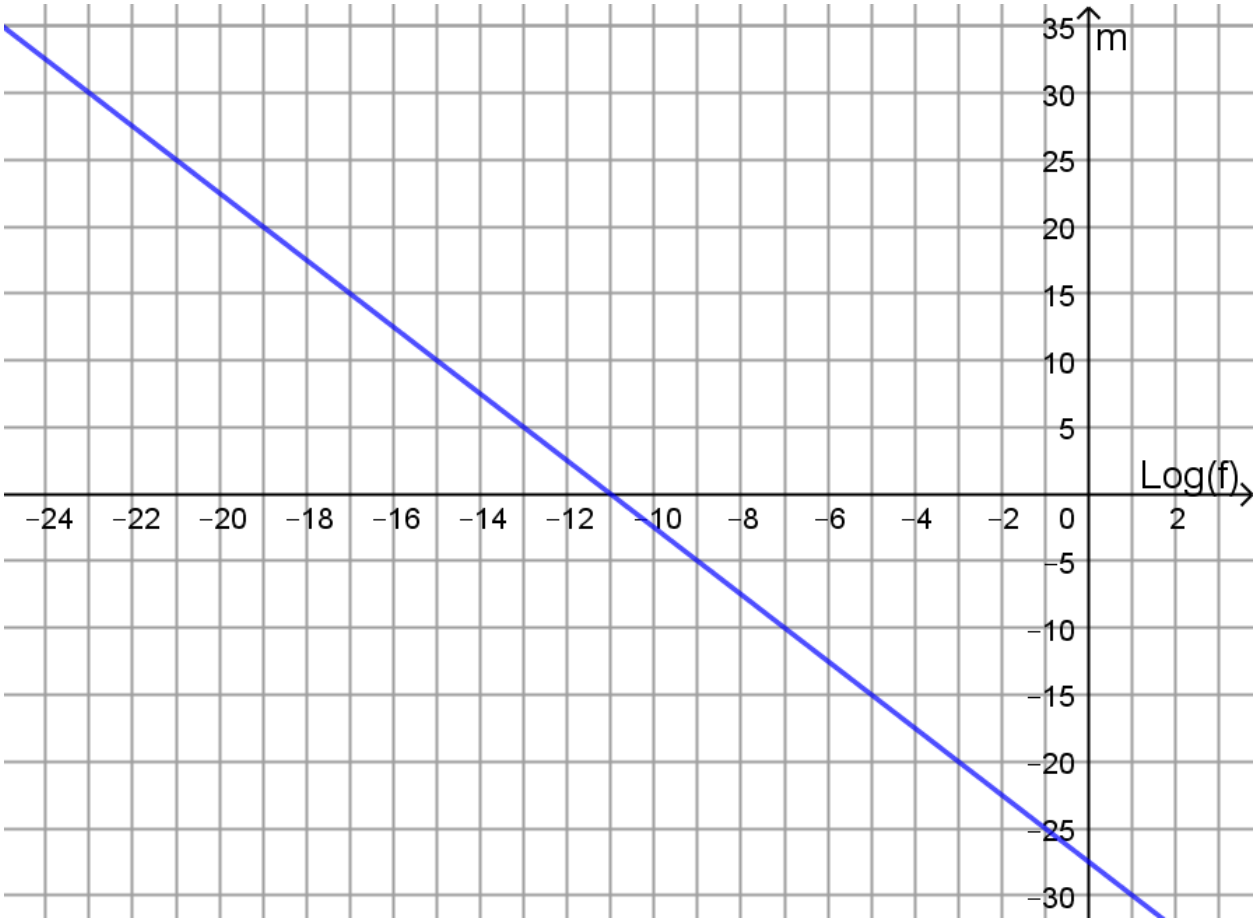
$$\mathbf{m = -27.5 - Log(f^{2.5})}$$

Students should recognise that laws of logs are required to convert the equation into:

m = - 27.5 - 2.5 Log(f) **M1**
where x-axes =Log(f) producing a linear equation

Gradient = -2.5 **A1**
y-intercept = -27.5 **A1**

[3]



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(iii) The Sun's apparent magnitude is -26.7 (to 3 s.f.) and the faintest stars visible to the naked eye have a approximately of approximately 6.5. Using the equation above, work out how much more 'brighter' (measured in flux) is the Sun that the faintest stars?

-26.7 = -27.5 – 2.5 Log(f_1) **M1** for setting up correctly the two equations

6.5 = -27.5 – 2.5 Log(f_2)

-26.7 – 6.5 = -27.5 – 2.5Log(f_1) - [-27.5 – 2.5 Log(f_2)] = - **2.5Log(f_1)** + 2.5 **Log(f_2)**

M1 set up and simplify correct subtraction equation

-20.2 = - **2.5 [Log(f_1) - Log(f_2)]**

$\frac{-20.2}{-2.5}$ = **8.08** = **Log**$\left(\frac{f_1}{f_2}\right)$ **M1** correct use of laws of logs

$10^{8.08}$ = $\frac{f_1}{f_2}$ therefore the **Sun** (f_1) = **$10^{8.08}$ brighter than the faintest star** (f_2) or 120 million flux brighter (to 3 s.f. (120 226 443.5))

(iv) Give a reason why the above function is undefined for f=0.

It's not possible to get a value of zero when taking the log (base 10) of a number i.e. no power of 10 will equal zero.

[1]

Q6) [3 marks]

If: $Log_{81}w = 2x - 1 \quad and \quad Log_w t = \frac{1}{2}$

Show that: $9^{2x} = 9t$ (you must show **all** steps of your working out, non-calc)

$Log_{81}w = 2x - 1 \quad and \quad Log_w t = \frac{1}{2}$

$81^{2x-1} = w$ $\qquad$ $w^{1/2} = t$ **M1 rewrite both log equations as power statements**

Substitute in 81^{2x-1} for 'w' in the second equation gives:

$(81^{2x-1})^{1/2} = t$ **M1 correct substitution**

Method 1

$(81^{1/2})^{2x-1} = t$ therefore: $(9)^{2x-1} = t$

⇨ $(9^2)^x \times 9^{-1} = \frac{9^{2x}}{9} = t$ multiplying both sides of equation by 9: **$9^{2x} = 9t$**

Method 2

$= (81^{2x} \times 81^{-1})^{1/2} = (81^{2x})^{1/2} \times (81^{-1})^{1/2} = 81^x \times 81^{-1/2}$

$= 81^x \times \frac{1}{\sqrt{81}}$ $=(9^2)^x \times 9^{-1} = \frac{9^{2x}}{9} = t$ **M1 simplification using laws of indices correctly – must be seen**

Multiplying both sides by 9: **$9^{2x} = 9t$**

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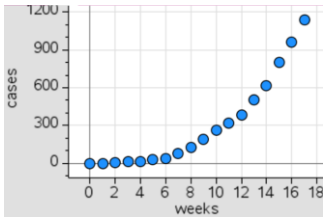
Q7) [5 marks]

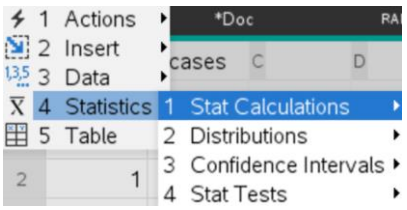
(a) Exponential: $y = (3.07691)1.47878^x$ **A1**

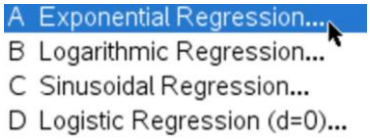
Logistic (d=0): $y = \frac{2116.21}{1+211.565e^{-0.323067x}}$ **A1**

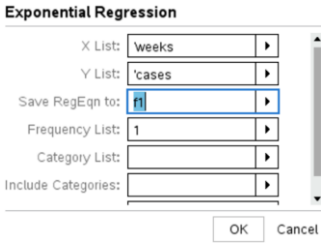
(ii) coefficient of determination (R^2) **A1 for exponential SSR and R^2**

	A weeks	B cases
=		
1	0	1
2	1	2
3	2	7
4	3	13
5	4	18

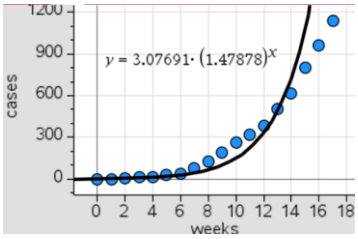








RegEqn	a*b^x
a	3.07691
b	1.47878
r^2	0.942566
r	0.970859



$\text{sum}(stat1.Resid^2)$ 2.10112E6

Ans: R^2 = 0.943 (to 3 s.f) **A1**

SSR = 2 101 120 ≈2 100 000 (to 3s.f.) **A1**

SSR is sensitive to the units making comparison across samples/data using different units difficult. R^2 effectively 'standardises' the units by dividing by the total variance in observed y-values. R^2 becomes a percentage of the total variation that is explained by the variation in the model's predicted y-values.

R1 any correct reference to the sensitivity to the units and how R^2 corrects for this.

[3]

(ii) Interpret the coefficient of determination for the exponential model.

For exponential model 94.3% of the variation in observed y-values is explained by the mode's predictions..

[1]

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