Mathematics: Analysis and Approaches first exams May 2021 (T.Garry - version 8)

SL*: common content; **SL**: only in Analysis course; **AHL**: additional HL content for Analysis course

■ Topic 1: Number and algebra

SL 1.1* Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and *k* is an integer.
SL 1.2* Arithmetic sequences and series; use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence; use of sigma notation for sums of arithmetic sequences; applications (e.g. simple interest); interpretation and prediction where a model is not perfectly arithmetic in real life.
SL 1.3* Geometric sequences and series; use of the formulae for the *n*th term and the sum of the first *n* terms of the sequence; use of sigma notation for the sums of geometric sequences; applications such as spread of disease, salary increase and decrease, population growth.
SL 1.4* Financial applications of geo. sequences and series including compound interest, annual depreciation.
SL 1.5* Laws of exponents with integer exponents; introduction to logarithms with base 10 and e; numerical evaluation of logarithms using technology.

SL 1.6 Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side proof; the symbols and notation for equality and identity.
SL 1.7 Laws of exponents with rational exponents; laws of logarithms; change of base of a logarithm; solving exponential equations, including using logarithms.
SL 1.8 Sum of infinite convergent geometric sequences.
SL 1.9 The binomial theorem; binomial expansions; use of Pascal's triangle and ${}^nC_r$ (compute using both the formula and technology).

AHL 1.10 Counting principles, including permutations and combinations; extension of the binomial theorem to fractional and negative indices.
AHL 1.11 Partial fractions; maximum of two distinct linear terms in the denominator, with degree of numerator less than the degree of the denominator.
AHL 1.12 Complex numbers; Cartesian form; the terms real part, imaginary part, conjugate, modulus and argument; the complex plane.
AHL 1.13 Modulus–argument (polar) form $z = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$; Euler form $z = re^{i\theta}$; sums, products and quotients in Cartesian, polar or Euler forms and their geometric interpretation.
AHL 1.14 Complex conjugate roots of quadratic and polynomial equations with real coefficients; De Moivre's theorem and its extension to rational exponents; powers and roots of complex numbers.
AHL 1.15 Proof by mathematical induction; proof by contradiction; use of a counterexample to show that a statement is not always true.
AHL 1.16 Solutions of systems of linear equations (a maximum of three equations in three unknowns), including cases where there is a unique solution, an infinite number of solutions or no solution.

■ Topic 2: Functions

SL 2.1* Different forms of the equation of a straight line; gradient; intercepts; parallel & perpendicular lines.
SL 2.2* Concept of a function, domain, range & graph; function notation; concept of a function as a math model; concept of inverse function; inverse function as a reflection in the line y=x; the notation $f^{-1}(x)$
SL 2.3* Graph of a function; creating a sketch from info given or a context, including transferring a graph from screen to paper; using technology to graph functions including their sums and differences.
SL 2.4* Determine key features of graphs; finding point of intersection of two curves or lines using technology

---

SL 2.5 Composite functions; identity function; finding the inverse function $f^{-1}(x)$
SL 2.6 Quadratic functions: graphs, y-intercept; axis of symmetry; factored form, x-intercepts; vertex form
SL 2.7 Solving quadratic equations and inequalities; using factorization, completing the square (vertex form); quadratic formula; discriminant and nature of roots: 2 distinct real roots, 2 equal real roots, no real roots.
SL 2.8 The reciprocal function 1/x: its graph and self-inverse nature; rational functions of the form $\frac{\text{linear}}{\text{linear}}$ and their graphs; equations of vertical and horizontal asymptotes.
SL 2.9 Exponential functions and their graphs; logarithmic functions and their graphs.

SL 2.10 Solving equations, both graphically and analytically; use of technology to solve a variety of equations, including those where there is no appropriate analytic approach; applications of graphing skills and solving equations that relate to real-life situations.
SL 2.11 Transformations of graphs; translations, reflections (in both axes), vertical stretches, horizontal stretches, composite transformations.

AHL 2.12 Polynomial functions, their graphs and equations; zeros, roots and factors; the factor and remainder theorems; sum and product of the roots of polynomial equations.
AHL 2.13 Rational functions of the forms $\frac{\text{linear}}{\text{quadratic}}$ and $\frac{\text{quadratic}}{\text{linear}}$; horizontal, vertical & oblique asymptotes
AHL 2.14 Odd & even functions; finding inverse function, including domain restriction; self-inverse functions.
AHL 2.15 Solutions of inequalities, both graphically and analytically.
AHL 2.16 Graphs of the functions $y = |f(x)|,\ y = f(|x|),\ y = \frac{1}{f(x)},\ y = f(ax+b),\ y = [f(x)]^2$; solution of modulus equations and inequalities.

■ Topic 3: Geometry and trigonometry

SL 3.1* The distance between two points in three-dimensional space, and their midpoint; volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids; the size of an angle between two intersecting lines or between a line and a plane.
SL 3.2* Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles; the sine rule (not including ambiguous case); the cosine rule; area of a triangle as $\frac{1}{2}ab\sin C$
SL 3.3* Applications of right and non-right-angled trigonometry, Pythagoras' theorem (contexts may include use of bearings); angles of elevation & depression; construction of labelled diagrams from written statements.

SL 3.4 The circle: radian measure of angles; length of an arc; area of a sector.
SL 3.5 Definition of sin*x* and cos*x* in terms of the unit circle; definition of tan*x* as sin*x* /cos*x*; exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples; extension of the sine rule to the ambiguous case.
SL 3.6 The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$; double angle identities for sine and cosine; the relationship between trigonometric ratios.
SL 3.7 The circular functions sin*x*, cos*x*, and tan*x*; amplitude, their periodic nature, and their graphs; composite functions of the form $f(x) = a\sin(b(x+c)) + d$; transformations; real-life contexts.
SL 3.8 Solving trigonometric equations in a finite interval, both graphically and analytically; equations leading to quadratic equations in sin*x*, cos*x*, or tan*x*.

AHL 3.9 Definition of the reciprocal trigonometric ratios; Pythagorean identities; the inverse trig functions; their domains and ranges; their graphs.
AHL 3.10 Compound angle identities; double angle identity for tan*x*.
AHL 3.11 Relationships between trigonometric functions and the symmetry properties of their graphs.
AHL 3.12 Concept of a vector; position vectors; displacement vectors; representation of vectors using directed line segments; base vectors ***i***, ***j***, ***k***; components of a vector; algebraic and geometric approaches to following: sum and difference of two vectors, zero vector, multiplication by a scalar, parallel vectors, magnitude of a vector, unit vectors, position vectors, displacement vector; proofs of geometrical properties using vectors.
AHL 3.13 The definition of the scalar product of two vectors; applications of the properties of the scalar product; the angle between two vectors; perpendicular vectors; parallel vectors.
AHL 3.14 Vector equation of a line in two and three dimensions; parametric form; Cartesian form; the angle between two lines; simple applications to kinematics.
AHL 3.15 Coincident, parallel, intersecting and skew lines, distinguishing between these cases; points of intersection.
AHL 3.16 The definition of the vector product of two vectors; properties of the vector product; geometric interpretations.
AHL 3.17 Vector equations of a plane: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$ and $\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n}$. Cartesian equation of a plane.
AHL 3.18 Intersections of: a line and a plane, 2 planes, 3 planes; angle between: a line and a plane, 2 planes.

■ Topic 4: Probability and statistics

SL 4.1* Concepts of population, sample, random sample, discrete and continuous data; reliability of data sources and bias in sampling; interpretation of outliers; sampling techniques: simple random, convenience, systematic, quota and stratified methods.
SL 4.2* Presentation of data (discrete and continuous); frequency histograms with equal class intervals; cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR); production and understanding of box and whisker diagrams; use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile range or range; determining whether data may be normally distributed by consideration of the symmetry of the box and whiskers.
SL 4.3* Measures of central tendency (mean, median and mode); estimation of mean from grouped data; modal class; measures of dispersion (interquartile range, standard deviation and variance); effect of constant changes on the original data; quartiles of discrete data.
SL 4.4* Linear correlation of bivariate data; Pearson's product-moment correlation coefficient, *r* ; scatter diagrams; lines of best fit, by eye, passing through the mean point; equation of the regression line of y on x; use of the equation of the regression line for prediction purposes; interpret the meaning of the parameters, *a* and *b*, in a linear regression.
SL 4.5* Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (*U*) and event; the probability of an event *A*; the complementary events; expected number of occurrences.
SL 4.6* Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities; combined events; mutually exclusive events; conditional probability; independent events.
SL 4.7* Concept of discrete random variables and their probability distributions; expected value (mean), E(*X*) for discrete data; applications.
SL 4.8* Binomial distribution; situations where the binomial distribution is an appropriate model; mean and variance of the binomial distribution.
SL 4.9* The normal distribution and curve; properties of normal distribution; diagrammatic representation; normal probability calculations; inverse normal calculations (no use of standardized normal variable *z*).

SL 4.10 Equation of the regression line of *x* on *y*; use of equation for prediction purposes (aware of reliability)
SL 4.11 Formal definition and use of the formulae: for conditional probabilities, and for independent events; testing for independence.
SL 4.12 Standardization of normal variables (*z*-values); inverse normal calculations where mean and standard deviation are unknown.

AHL 4.13 Use of Bayes' theorem for a maximum of three events.
AHL 4.14 Variance of a discrete random variable; continuous random variables and their probability density functions. including piecewise functions; mode and median of continuous random variables; mean, variance and standard deviation of discrete and continuous random variables; the effect of linear transformations.

■ Topic 5: Calculus

SL 5.1* Introduction to concept of a limit; derivative interpreted as gradient function and as rate of change.
SL 5.2* Increasing and decreasing functions: graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$
SL 5.3* Power rule for differentiation; derivative of functions of the form $f(x) = ax^n + bx^{n-1} \ldots$
SL 5.4* Tangents and normals at a given point, and their equations.
SL 5.5* Introduction to integration as anti-differentiation; definite integrals using technology; areas between a curve and the x-axis; anti-differentiation with a boundary condition to determine the constant term.

SL 5.6 Derivative of $x^n \ (n \in \mathbb{Q}), \sin x, \cos x, e^x$ and $\ln x$; differentiation of a sum and a multiple of these functions; the chain rule for composite functions; the product and quotient rules.
SL 5.7 The second derivative; graphical behaviour of functions, including the relationship between the graphs of a function, its 1st derivative and its 2nd derivative.
SL 5.8 Local maximum and minimum points; testing for maximum and minimum; optimization; points of inflexion with zero and non-zero gradients.
SL 5.9 Kinematic problems involving displacement, velocity, acceleration and total distance travelled.
SL 5.10 Indefinite integral of $x^n \ (n \in \mathbb{Q}), \sin x, \cos x, \frac{1}{x}$ and e^x ; composites of any of these with the linear ites function ax+b; integration by inspection or by substitution for expressions of the form $\int kg'(x)f(g(x))\mathrm{d}x$

SL 5.11 Definite integrals, including analytical approach; areas of a region enclosed by a curve and the -axis, where the curve can be positive or negative, without the use of technology; areas between curves.

AHL 5.12 Informal understanding of continuity and differentiability of a function at a point; understanding of limits (convergence and divergence); definition of derivative from first principles; higher derivatives.
AHL 5.13 The evaluation of limits of the form and using l'Hôpital's rule; repeated use of l'Hôpital's rule.
AHL 5.14 Implicit differentiation; related rates of change; optimisation problems.
AHL 5.15 Derivatives of all six trig functions and their inverses; indefinite integrals of the derivatives of any of these functions; composites of any of these with a linear function; use partial fractions to rearrange integrand
AHL 5.16 Integration by substitution; integration by parts; repeated integration by parts.
AHL 5.17 Area of the region enclosed by a curve and the *y*-axis in a given interval; volumes of revolution about the *x* -axis or *y* -axis.
AHL 5.18 First order differential equations; numerical solution of using Euler's method; by separation of variables; homogeneous differential equations; solutions using the integrating factor method.
AHL 5.19 Maclaurin series; use of simple substitution, products, integration and differentiation to obtain other series; Maclaurin series developed from differential equations.